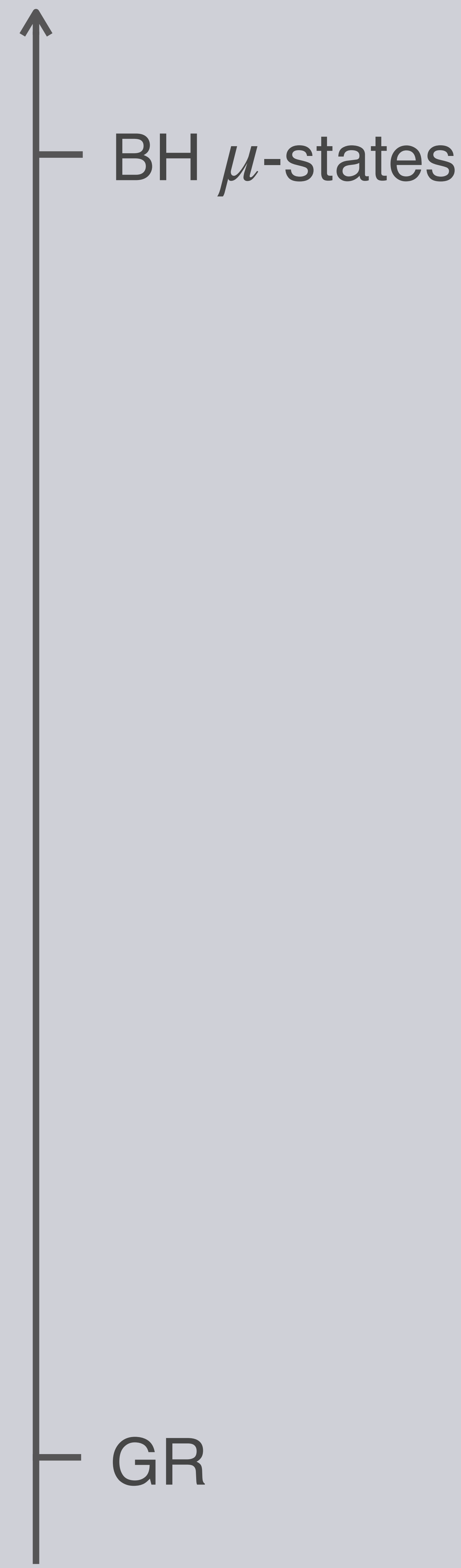
holography

chaos $\hookrightarrow$ quantum gravity



Late time chaos in 2d gravity

Yerevan, Jun. 23

Alexander Altland (Cologne),
Julian Sonner (Geneva),
Boris Post, Jeremy van der Hayden, Erik Verlinde (Amsterdam)

quantum chaos review

chaos in semiclassical gravity

chaos in string field theory

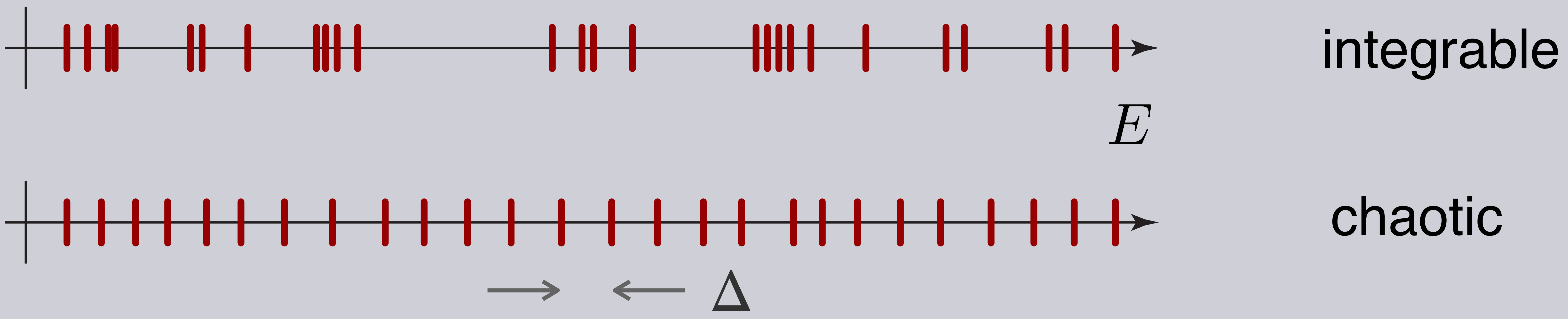
arXiv:2008.02271, SciPost. 22
arXiv:2204.07583, SciPost 23

chaos (review)

phenomenological signatures

- states uniformly Gaussian distributed in Hilbert space (max. entropy, cf. ETH)

- spectrum shows high degree of order



quantitatively described by

$$R_2(\omega) \equiv \frac{1}{\Delta^2}\langle \rho(E+\omega)\rho(E)\rangle \quad \longrightarrow \quad R_2(s) \qquad \boxed{s = \frac{\pi\omega}{\Delta}}$$

Spectral correlation function

small energies, $s \lesssim 1$, deep quantum regime

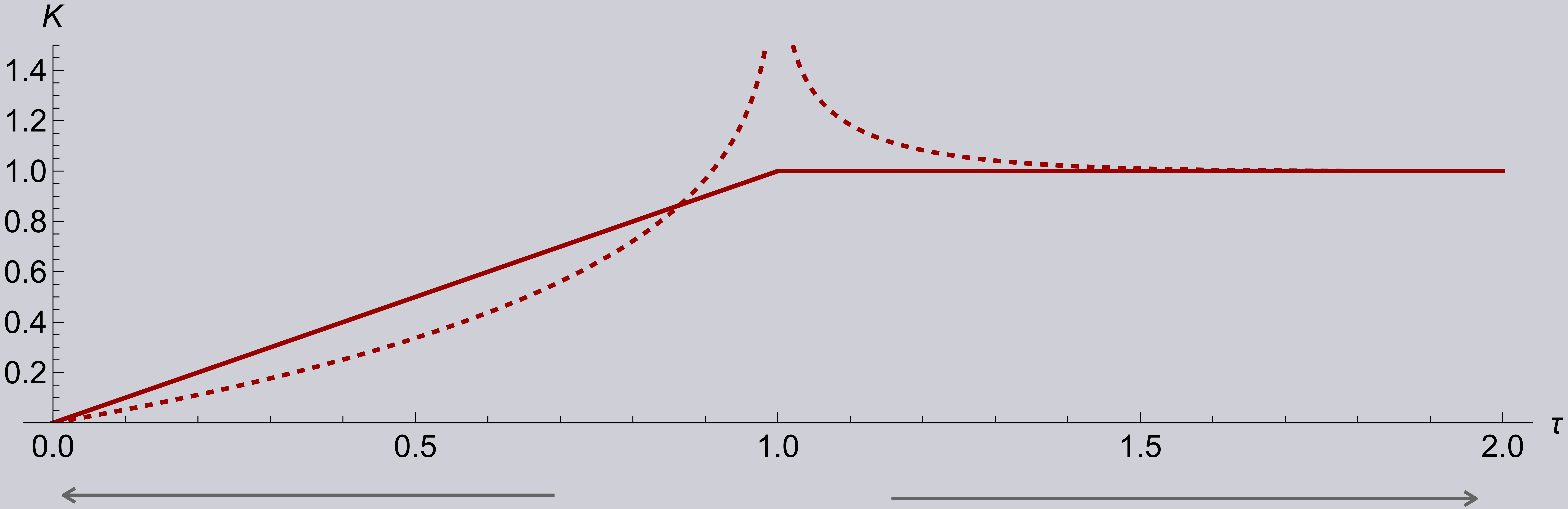
large energies, $s \gg 1 (\omega \gg \Delta)$, semiclassical regime

- level repulsion: $R_2(0) = -1$
- level crystallization: oscillations
- decay of correlations in powers of s^{-1}

*) Detailed structure depends on symmetry class: A, AI, AII, AIII, BDI, C, CI, CII, D, DIII.

AA, Zirnbauer, 97

Spectral form factor



short times, $\tau \ll 1$, semiclassical regime

long times, $\tau \gtrsim 1$, deep quantum regime

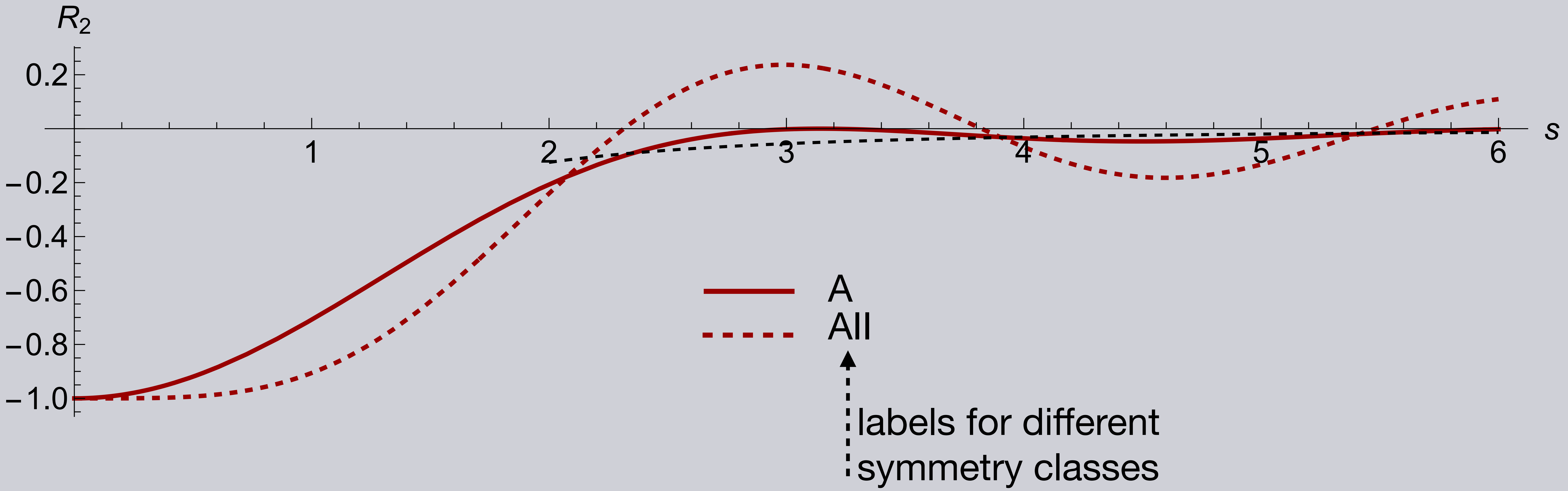
Spectral correlation function

small energies, $s \lesssim 1$, deep quantum regime

large energies, $s \gg 1 (\omega \gg \Delta)$, semiclassical regime

- level repulsion: $R_2(0) = -1$
- decay of correlations in powers of s^{-1}
- level crystallization: oscillations

spectral rigidity cont'd



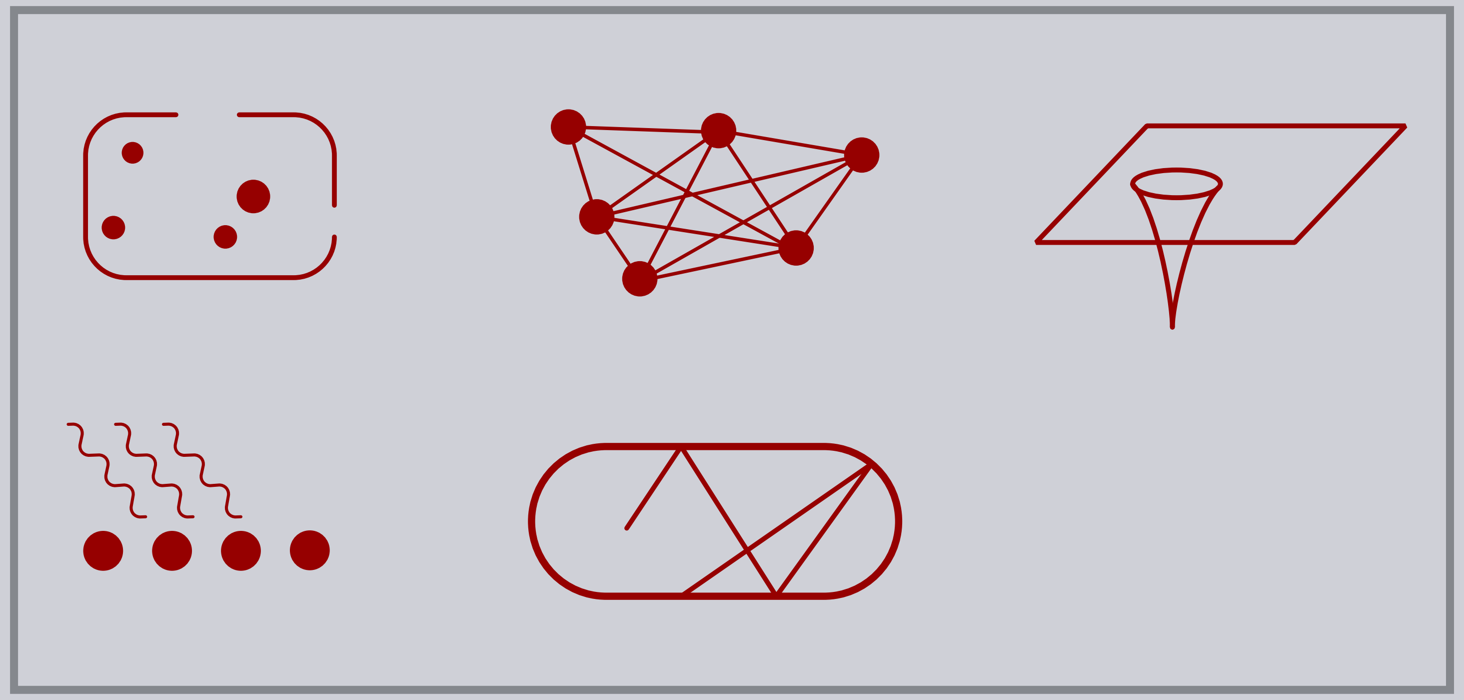
$$R_2(\omega) \equiv \frac{1}{\Delta^2}\langle \rho(E+\omega)\rho(E)\rangle$$

$$R_2(s) = -\frac{\sin^2(s)}{s^2}$$

A (GUE)

$$R_2(s) = \frac{\sin^2(2s)}{(2s)^2} - \frac{d}{d(2s)}\frac{\sin(2s)}{2s}\int_0^1 \frac{\sin(2st)}{t}$$

AII (GSE)



Shown by chaotic systems in cond-mat, AMO, …, gravity.

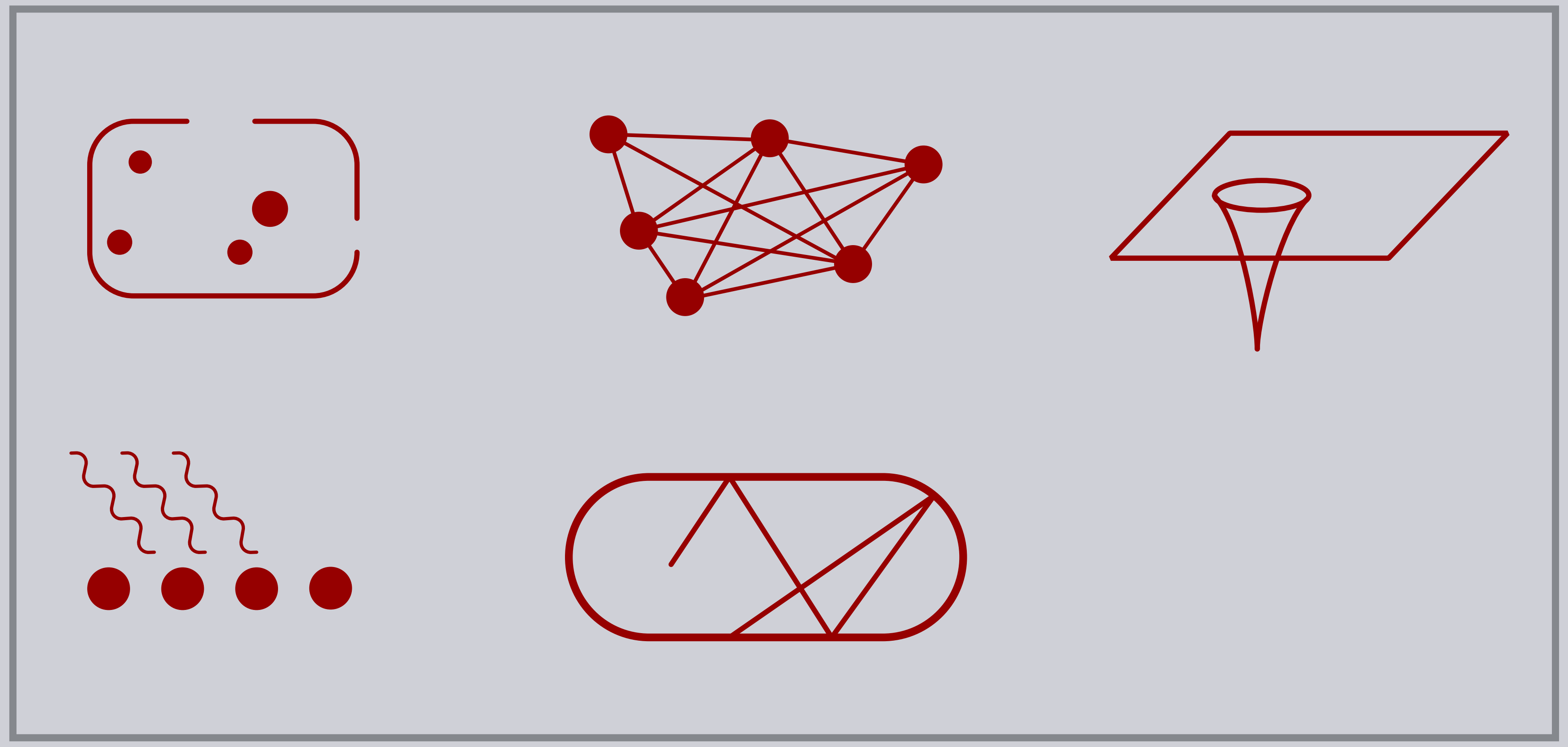
Q: How can this level of universality be understood?

Understanding universal spectral correlations

Constructive approach (aka Bohigas–Gianonni–Schmit (GGS) conjecture)

Wigner 1955

RMT $\boxed{H = \{H_{\mu\nu}\}, \qquad \mathrm{var}(H_{\mu\nu}) = \mathrm{const}\,.}$

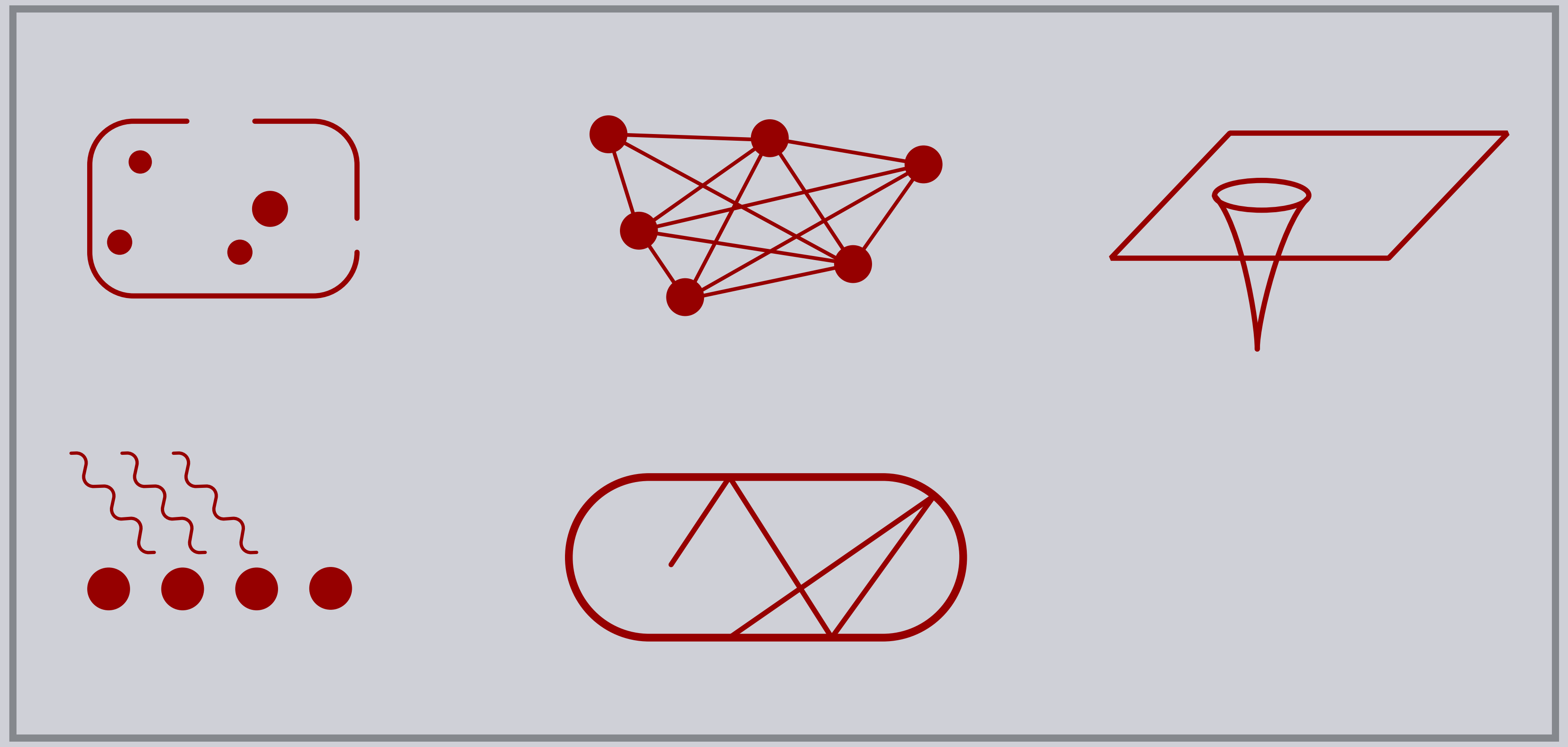


chaotic systems in cond-mat, AMO, …, gravity.

$\xrightarrow[\text{long times}]{\text{low energies}}$ universal correlations

Understanding universal spectral correlations

Conceptual approach



chaotic systems in cond-mat, AMO, …, gravity.

$\xrightarrow[\text{long times}]{\text{low energies}}$ $\boxed{\text{effective theory}}$

Wegner 1980
Efetov 1981

cf. “mean field theory of magnetism”

Understanding universality (from RMT)

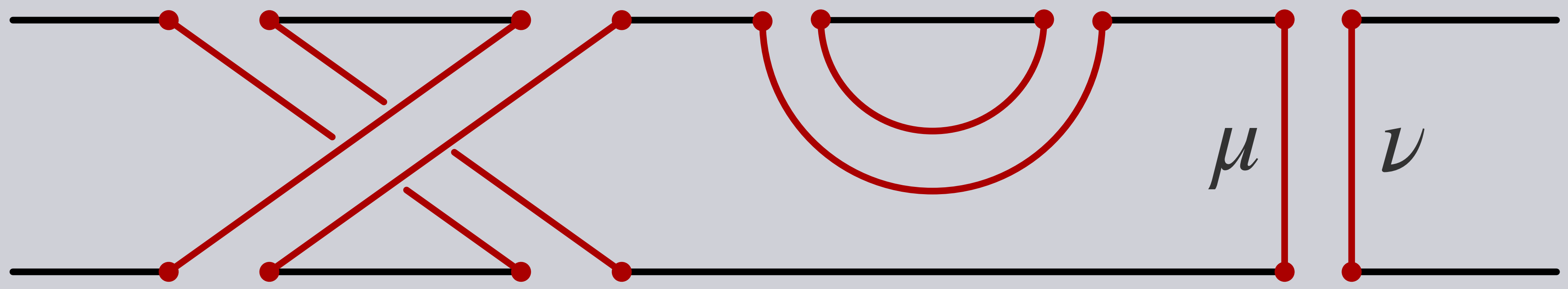
- consider spectral correlation function

$$R_2(\omega) = \frac{1}{\Delta^2}\langle \rho(E+\omega)\rho(E)\rangle \;\rightarrow\; \langle G^+G^-\rangle$$

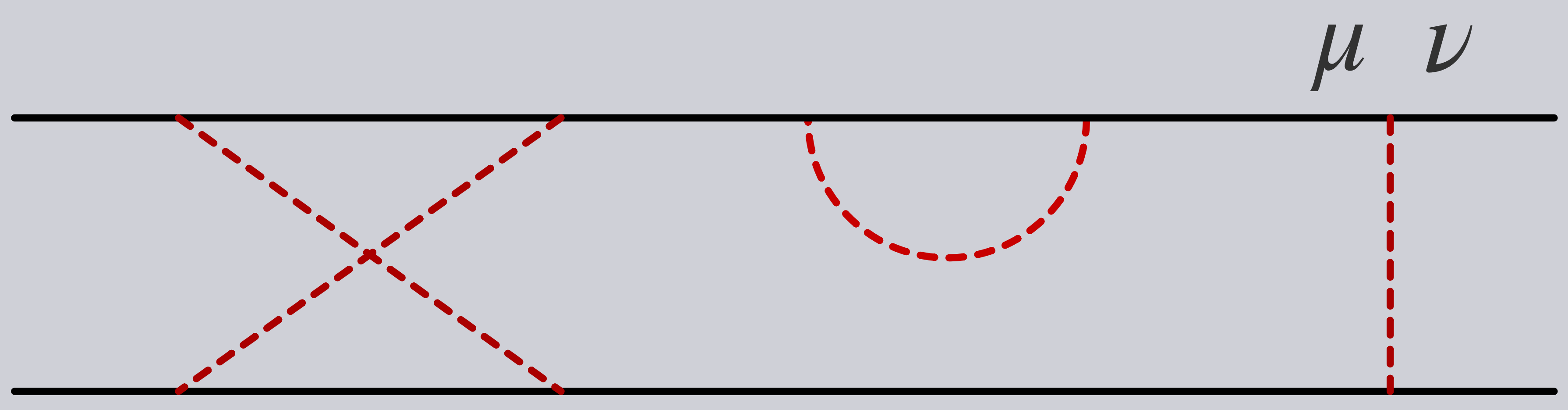
- for RMT Hamiltonian

$$H = \{H_{\mu\nu}\}, \qquad \mathrm{var}(H_{\mu\nu}) = \mathrm{const.}, \qquad \dim(H) = D$$

- perturbative expansion . . .



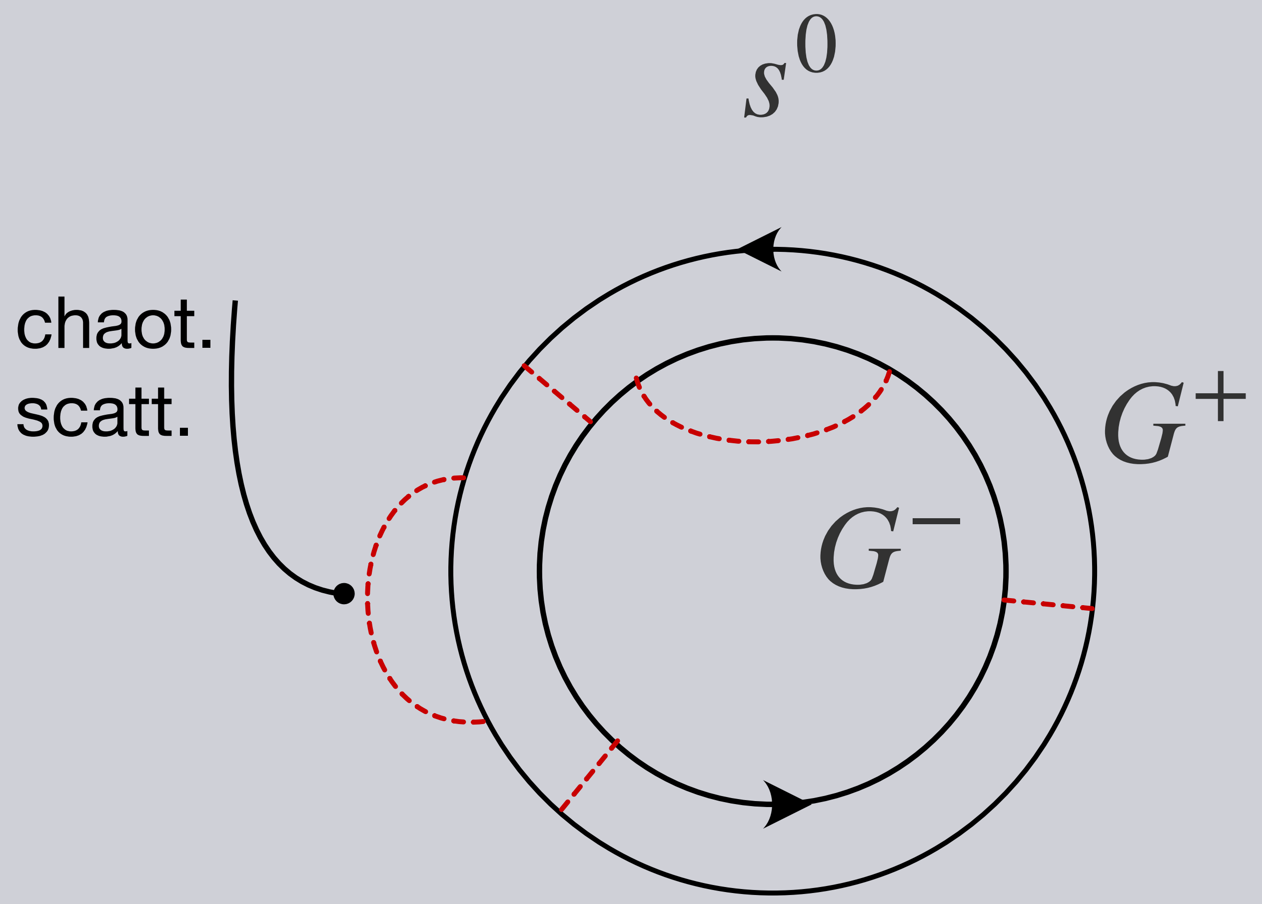
hep-th: “double line notation”



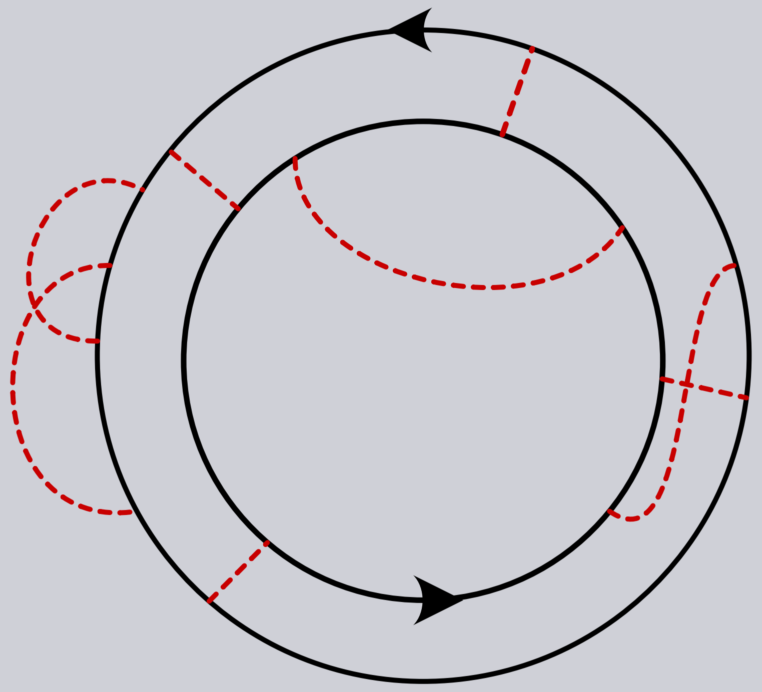
cond-mat: “impurity diagram notation”

Perturbative expansion I: topological recursion

$g = 0$



$g > 0$



topological expansion for 2-point function:

$$\langle G^+ G^- \rangle = \sum_g D^{-2g} R_{g,2}$$

topological recursion (symbolically):

Eynard-Orantin, 07

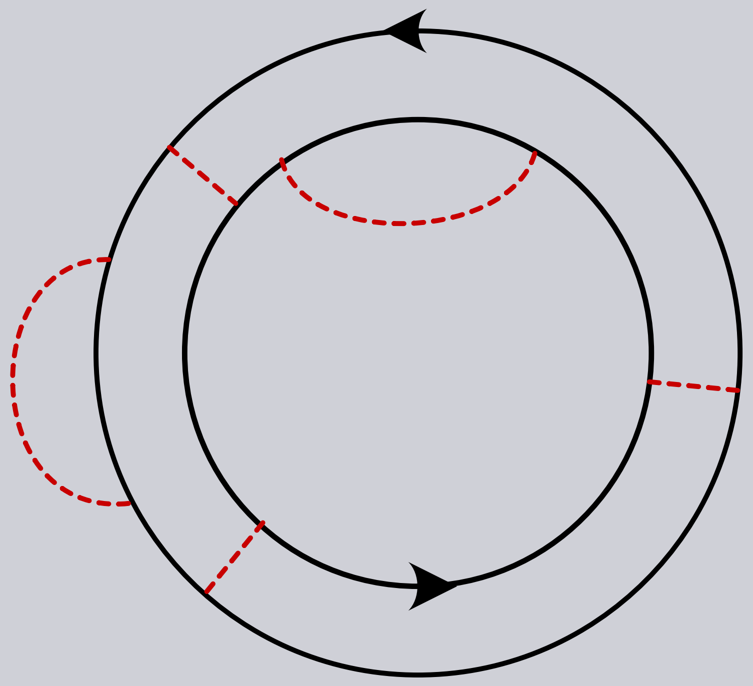
$$R_{g,n} = \mathscr{F}(R)$$

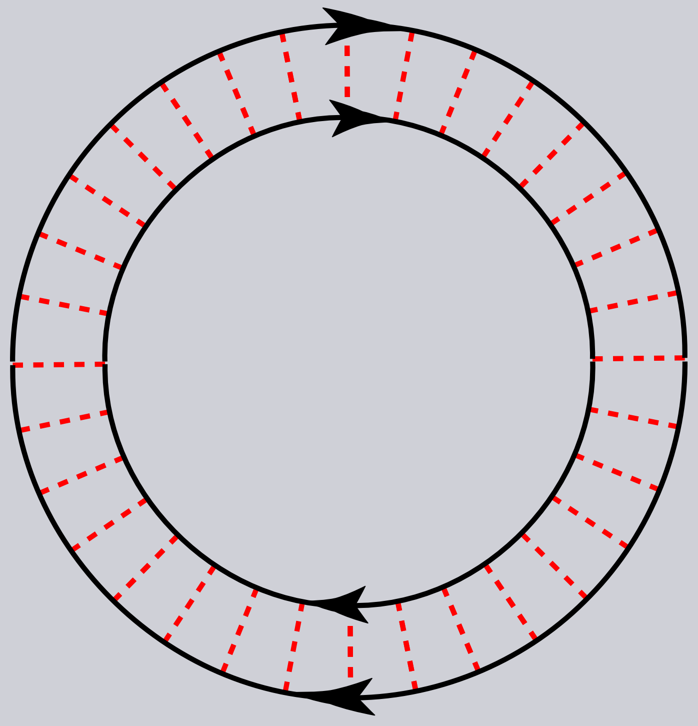
$$R = \{R_{h,n} \,|\, n = 1,2,3; h \leq g\}$$

Perturbative expansion II: g vs. $1/s$ expansion

s^0 | $s^{-(g+1)}$

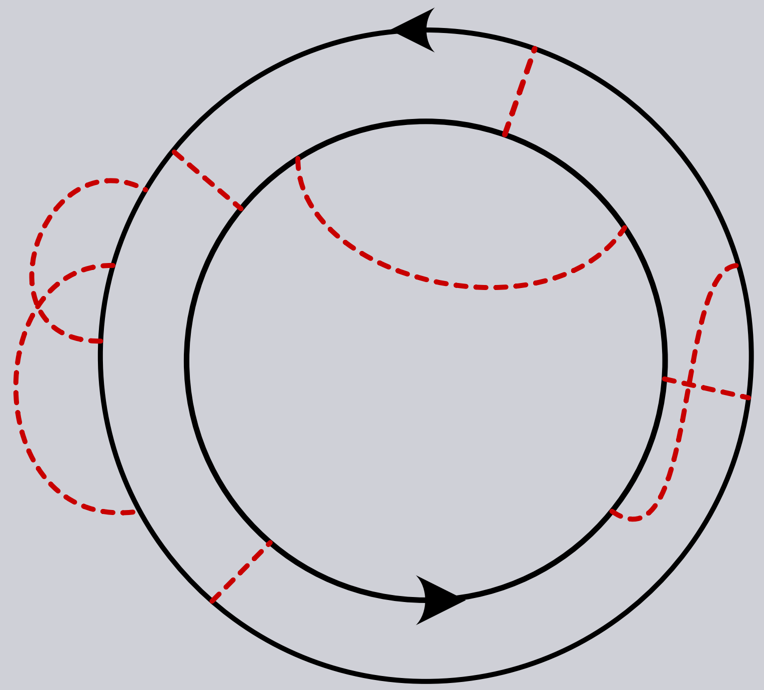
$g = 0$

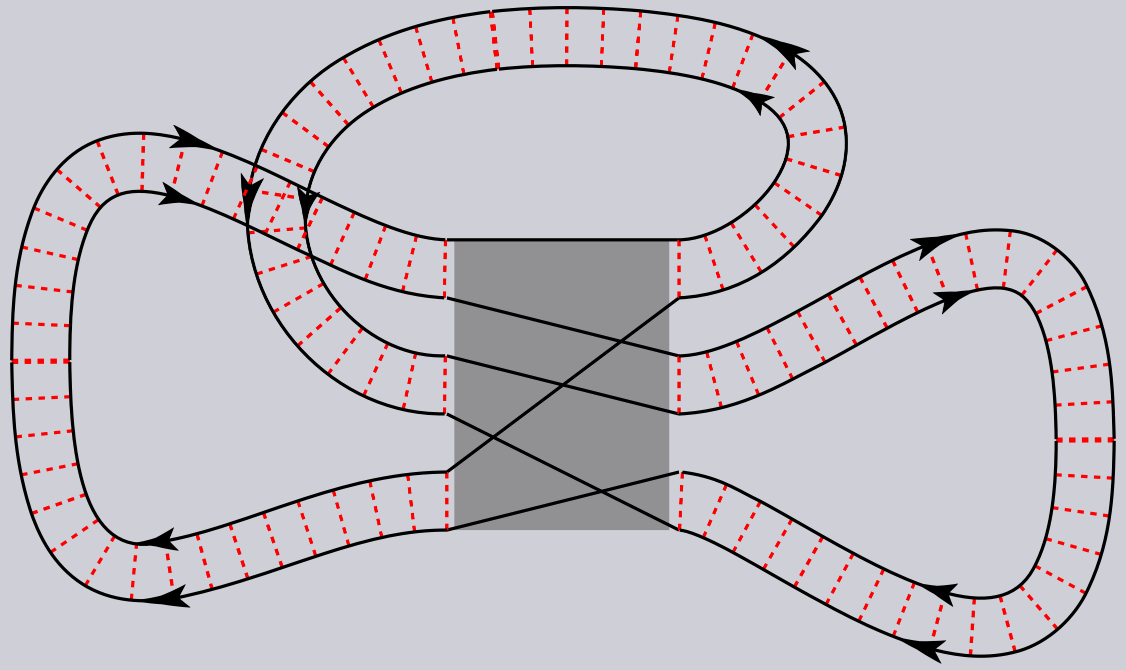




Berry, 85
Altshuler & Shklovskii, 86

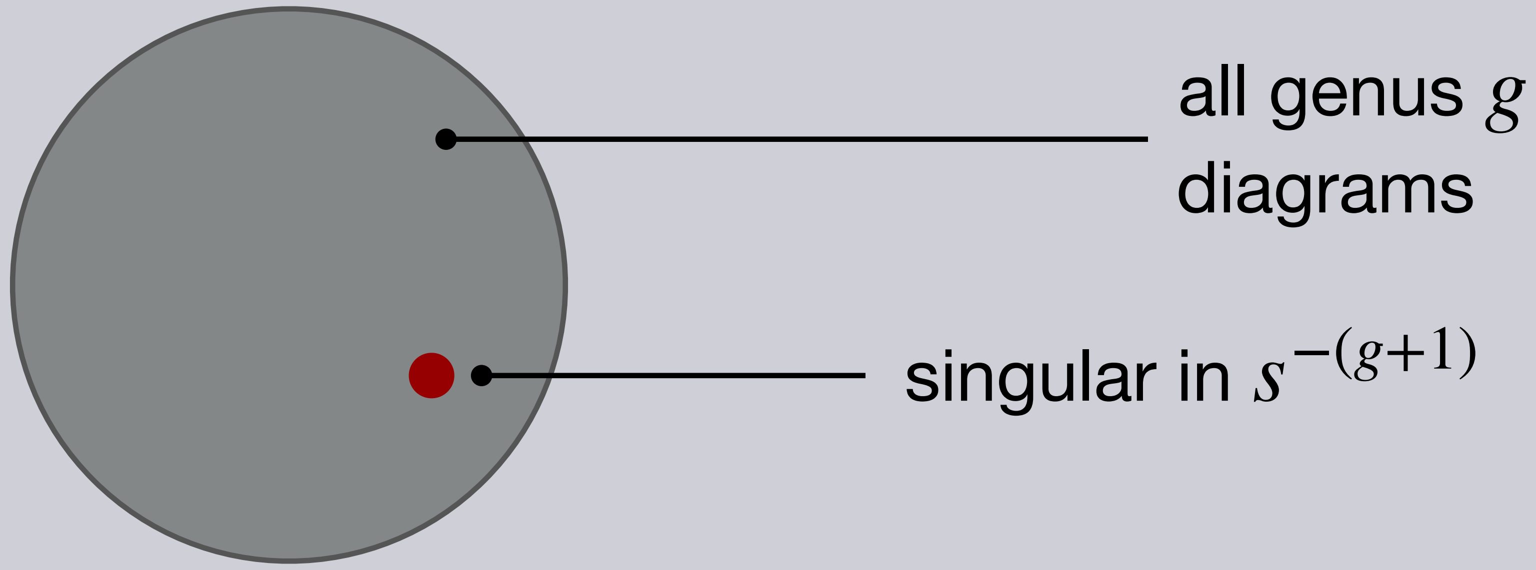
$g > 0$





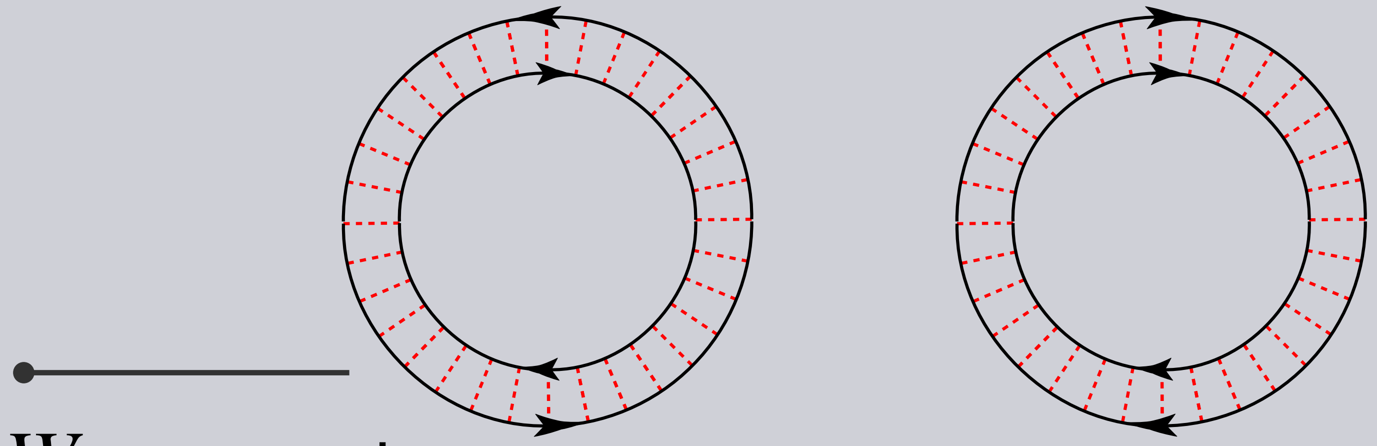
cf. Aleiner & Larkin, 96
cf. Sieber & Richter, 01

- singular diagrams: ‘small’ subset of $R_{g,2}$
- degree of singularity set by g

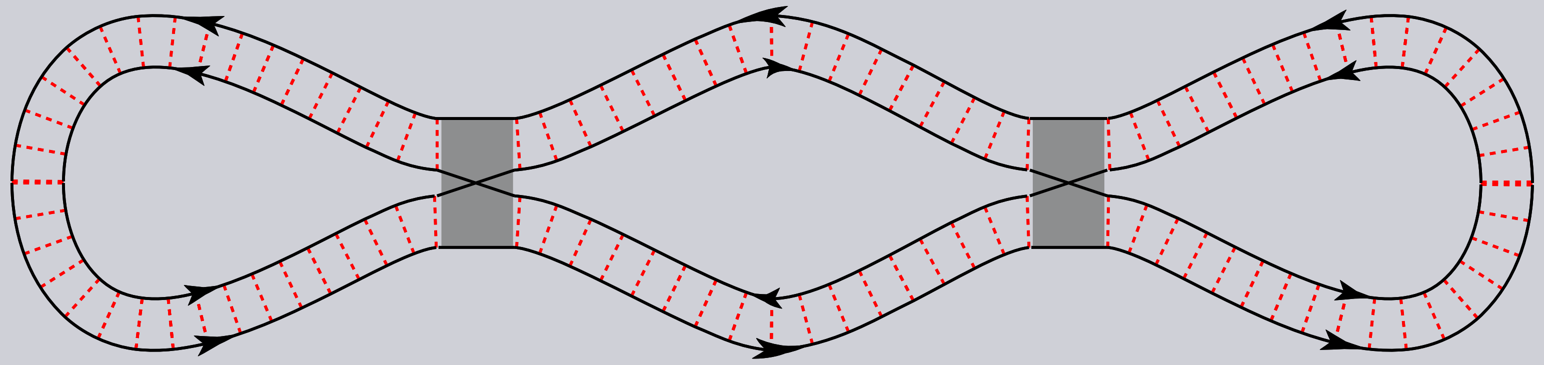


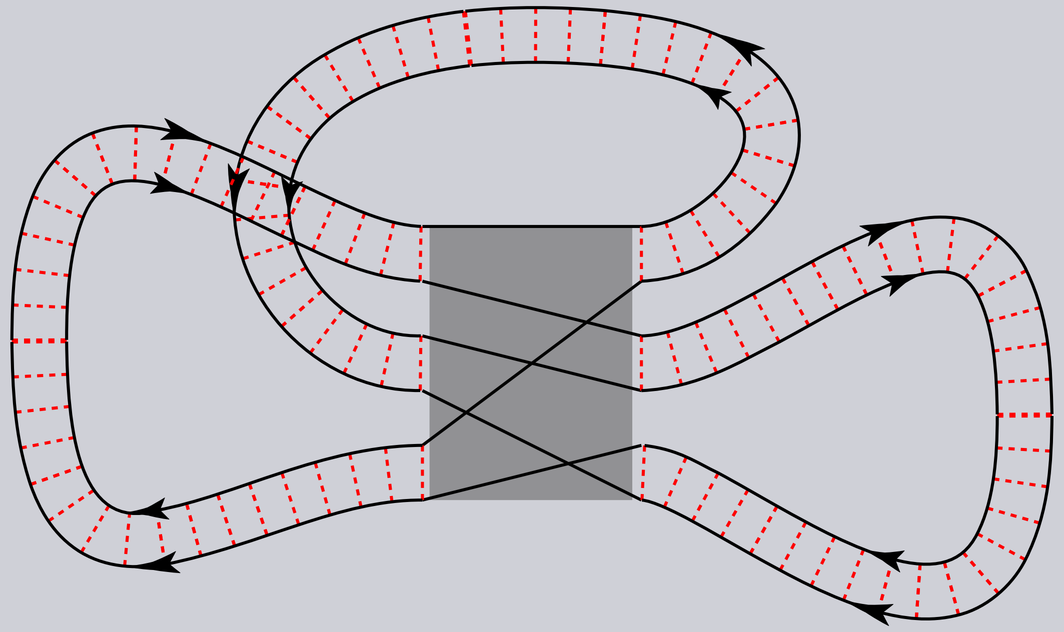
Perturbative expansion III: field theory

$$Q = T\tau_3 T^{-1}, \quad T = \exp(W)$$



W-propagator
$\sim s^{-1}$





$$\boxed{Z = \int_{X(n|n)} dQ\, e^{is\,\mathrm{tr}(Q\tau_3)}}$$

Wegner 1980
Efetov 1981

effective theory of
ergodic quantum chaos

- Q: low dimensional (flavor) matrices
- diagrams: loop expansion
- full integration: correlation functions beyond perturbation theory

chaos in semiclassical gravity

Holography background

The holographic principle: d-dimensional gravitational systems cast $(d-1)$-dimensional holographic shadows.*

Black holes are chaotic systems.

't Hooft 93, Susskind 95

$>$ 2015 search for a simple dimensional holographic correspondence between 1-dimensional quantum chaotic boundary theory and 2-dimensional gravity theory.

Kitaev 15, …

* classic example: gravity in $\mathrm{AdS}_5 \times S^5$ $(d = 5) \rightarrow \mathcal{N} = 4$ super Yang-Mills $(d = 4)$

Maldacena 97

Sachdev-Ye-Kitaev Model (15)

A model of N randomly interacting *Majorana* fermions

$$\hat{H} = \sum_{ijkl}^{N} \underbrace{J_{ijkl}}_{\text{random}} \hat{\chi}_i \hat{\chi}_j \hat{\chi}_k \hat{\chi}_l, \qquad \{\hat{\chi}_i, \hat{\chi}_j\} = 2\delta_{ij}$$

Kitaev 15

cf. Sachdev, Ye 90

cf. Bohigas, French, Weidenmüller, … early 70s

SYK model

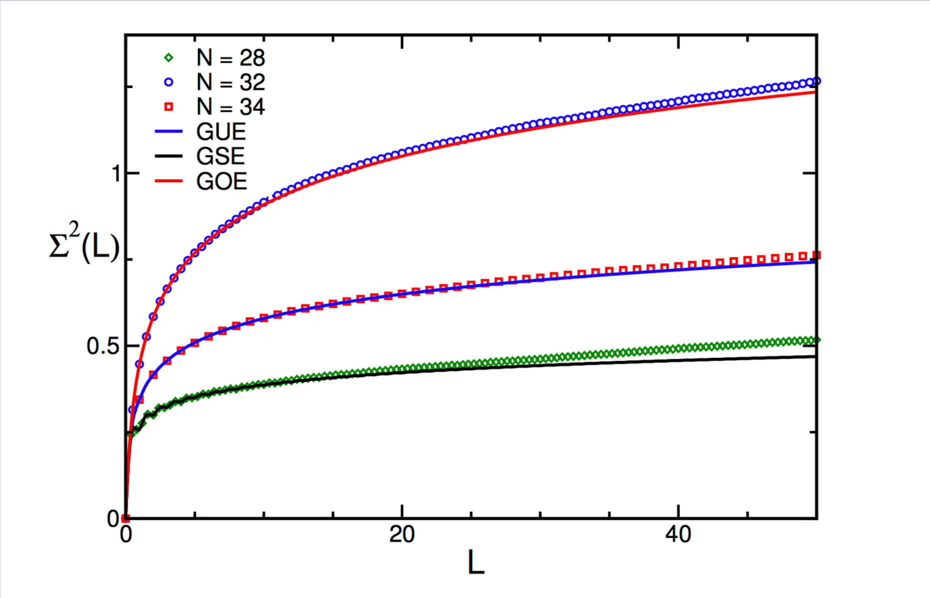
- ☑ one-dimension (quantum mechanics)
- ☑ hard quantum chaos

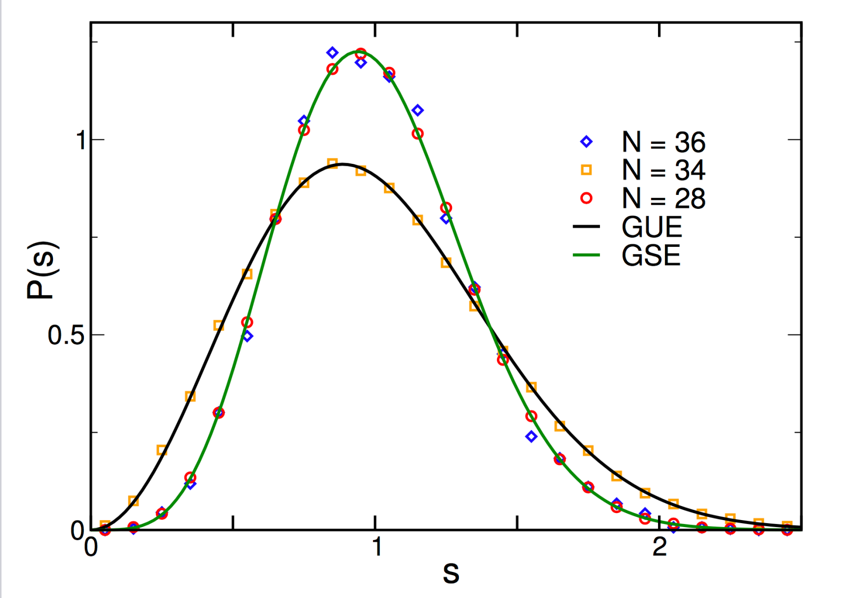
SYK – quantum chaos

ergodic spectral correlations witnessed by correlation functions

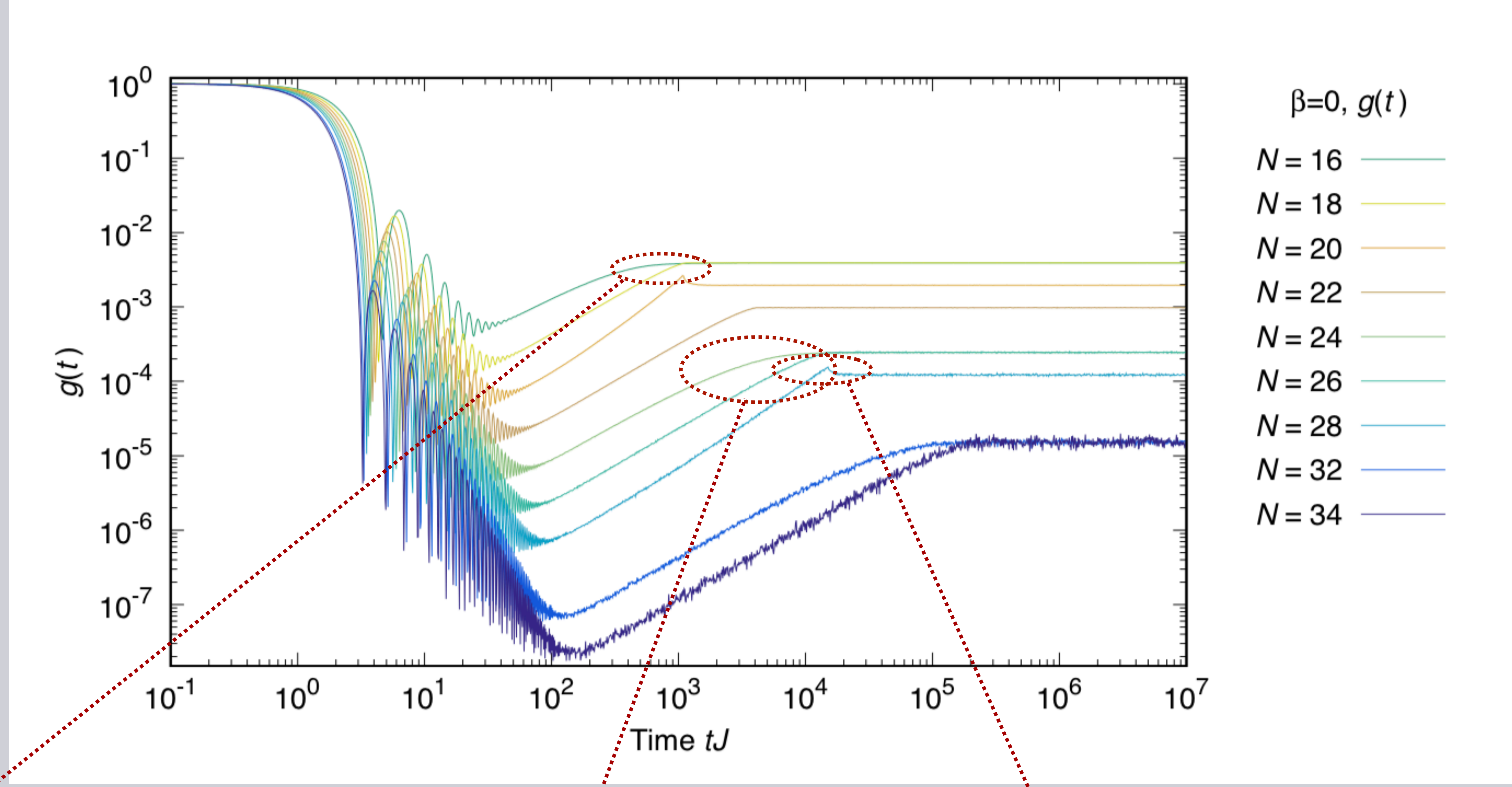
effective field theory identified

Bagrets, aa, 18

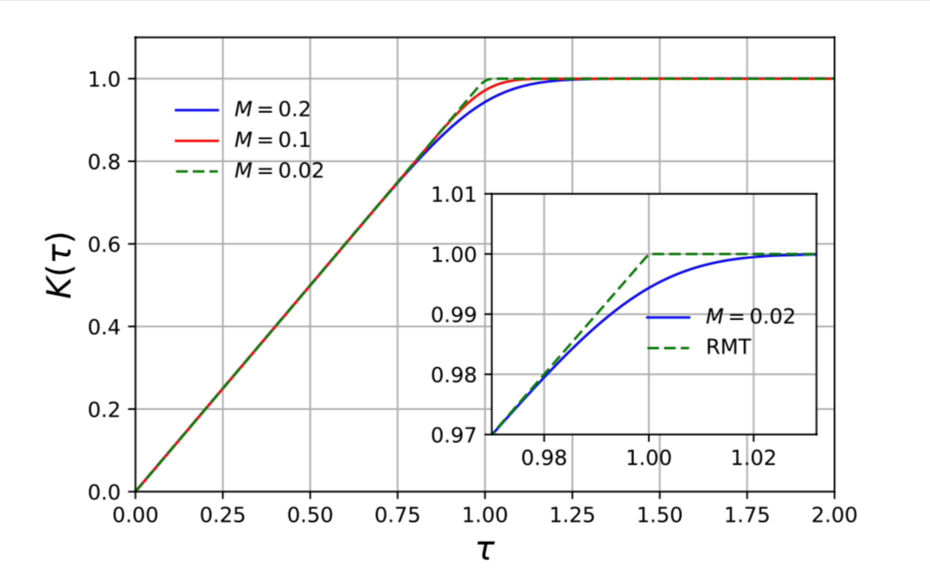


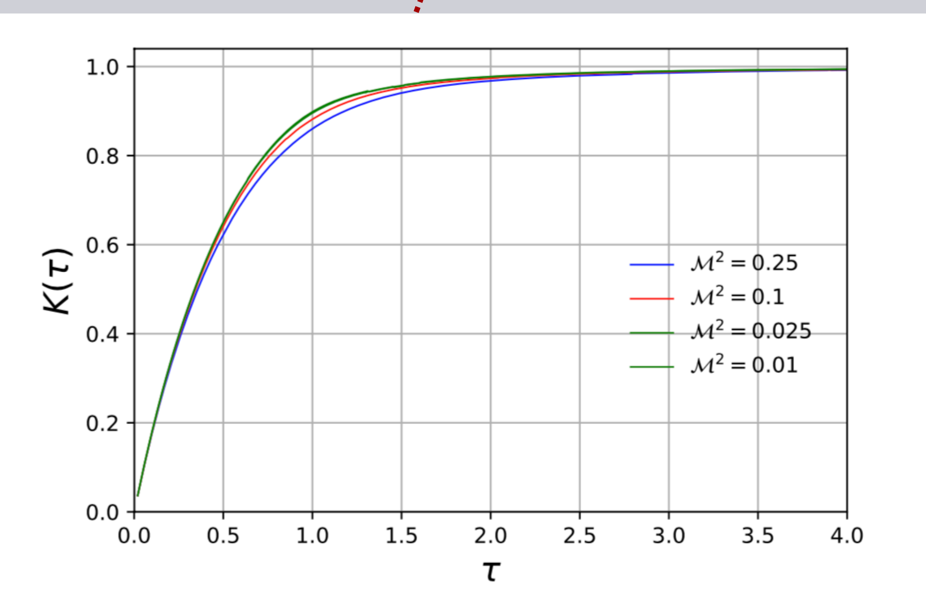


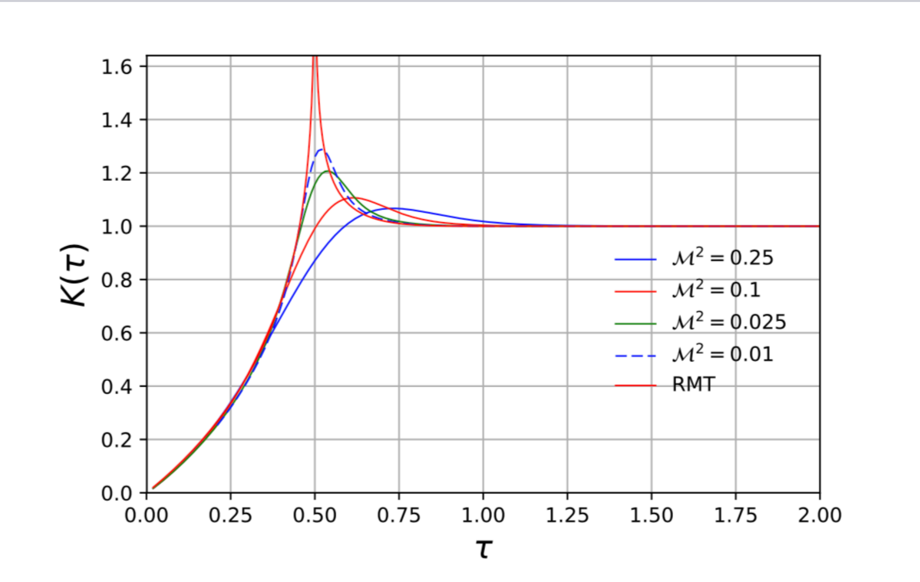
Verbaarschot, Garcia-Garcia, 16



Cotler et al. 17







aa, Micklitz, Verbaarschot unpublished

SYK holographic correspondence

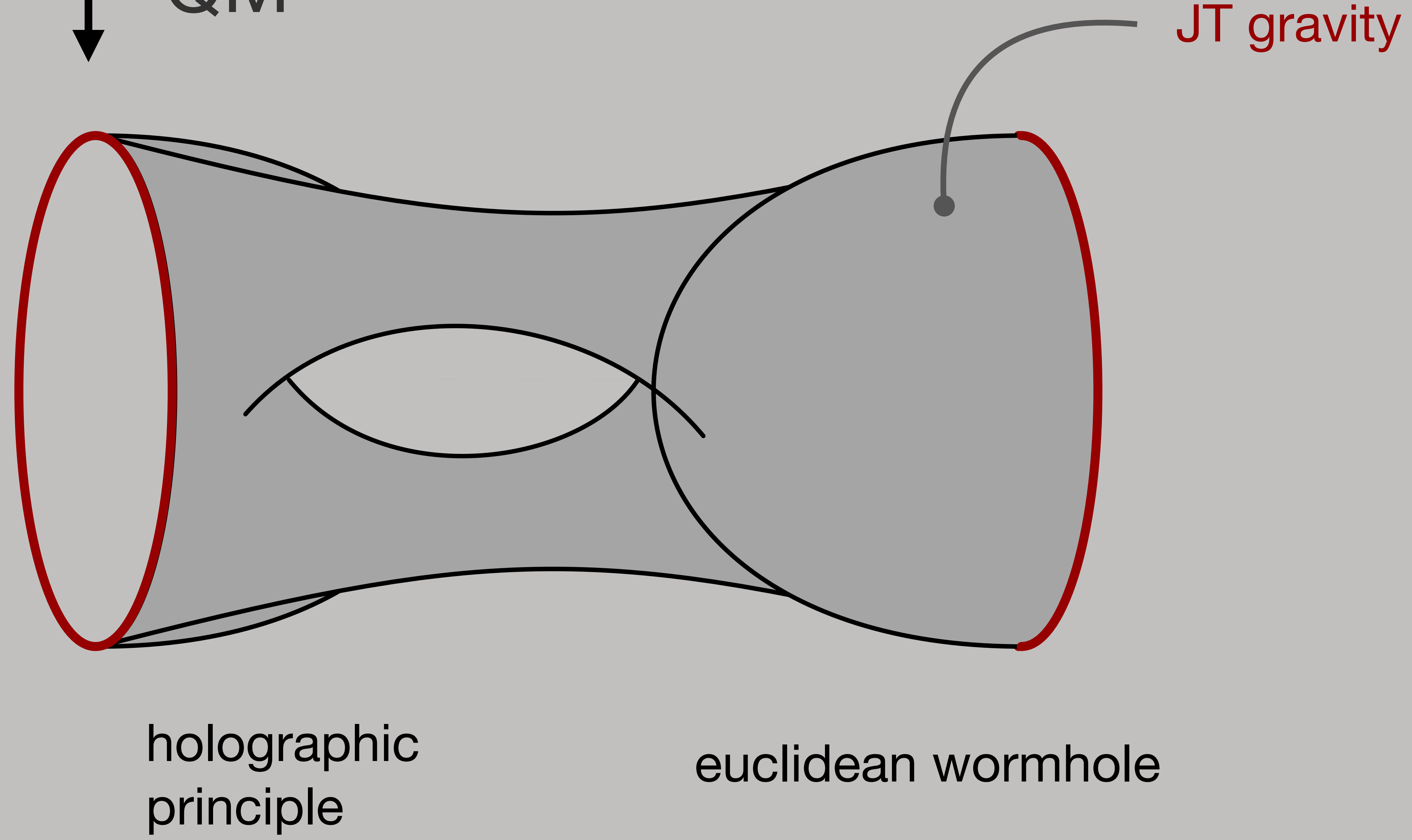
$$\left\langle\; G = \frac{1}{z-H} \qquad\qquad G = \frac{1}{z-H} \;\right\rangle_{\text{disorder}}$$

individual systems (?)

$\updownarrow$ imag. time FT

$$\left\langle\; Z = \operatorname{tr} e^{-\beta H} \qquad\qquad Z = \operatorname{tr} e^{-\beta H} \;\right\rangle$$

$\updownarrow$ imag. time QM



JT gravity

Jackiw, 83
Teitelboim, 85

2d Einstein–Hilbert action coupled to dilaton field

dilaton field

negative cosmological constant

$$S = \frac{1}{16\pi G}\int \sqrt{g}\,\phi(R + \Lambda) + \ldots$$

curvature

Jackiw Teitelboim gravity


time
$\sim 2^N$
$\sim N$
SYK
JT
g/s^{-1} – expansion
Schwarzian action
Eynard, Orantin, 08
Mirzhakani, 06
Kitaev, Suh,
Stanford,
Maldacena, aa,
Bagrets, …
2015+

The gravitational path integral

JT partition sum

topological action

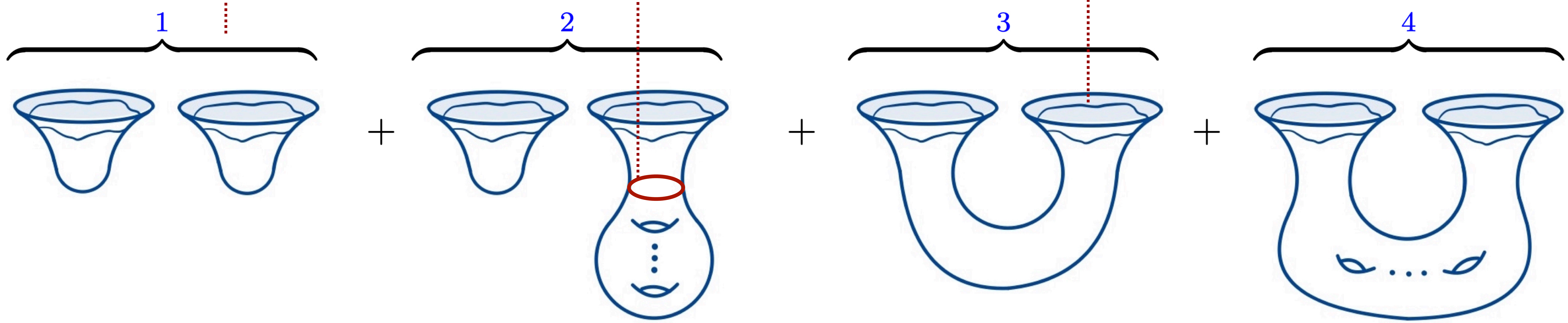
Schwarzian action

$$Z = \sum_g e^{-S_0 g} \int (\text{moduli space}) \int (\text{boundary wiggles})\, e^{-\int_{\text{boundary}} \mathscr{K}\phi}$$

genus expansion

these parameters

"an integral over geometries …"

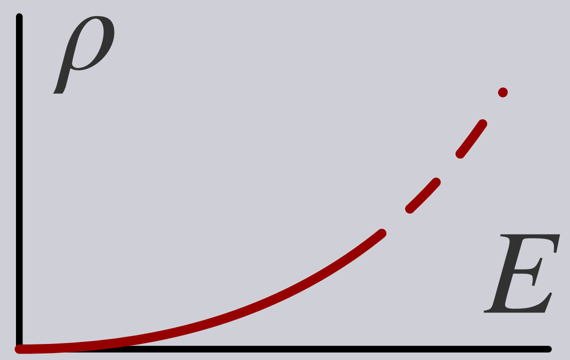


Post, v.d. Heyden, Verlinde, 22

topological recursion (gravity)

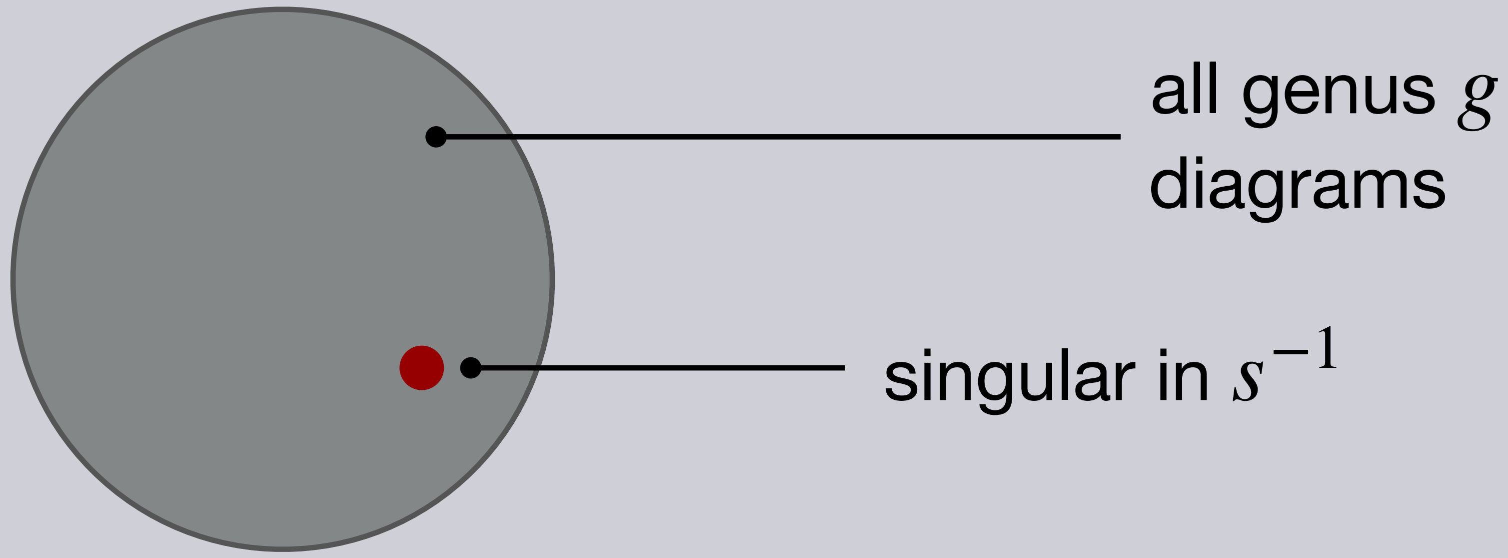
I: Formulate recursive relation for topological expansion of JT — Mirzhakani, 06

II: compare to topological recursion of matrix model with boundary spectral density fine tuned to match that of SYK. — Eynard-Orantin, 07



$\rightarrow$ perfect match

Expansion of JT partition sum captures perturbative expansion of spectral correlations



Shenker *et al.* 19
aa, Sonner, 21

Q: Where is the non-perturbative part of spectral correlations? — cf. Saad *et al.* 22

A: In string theory

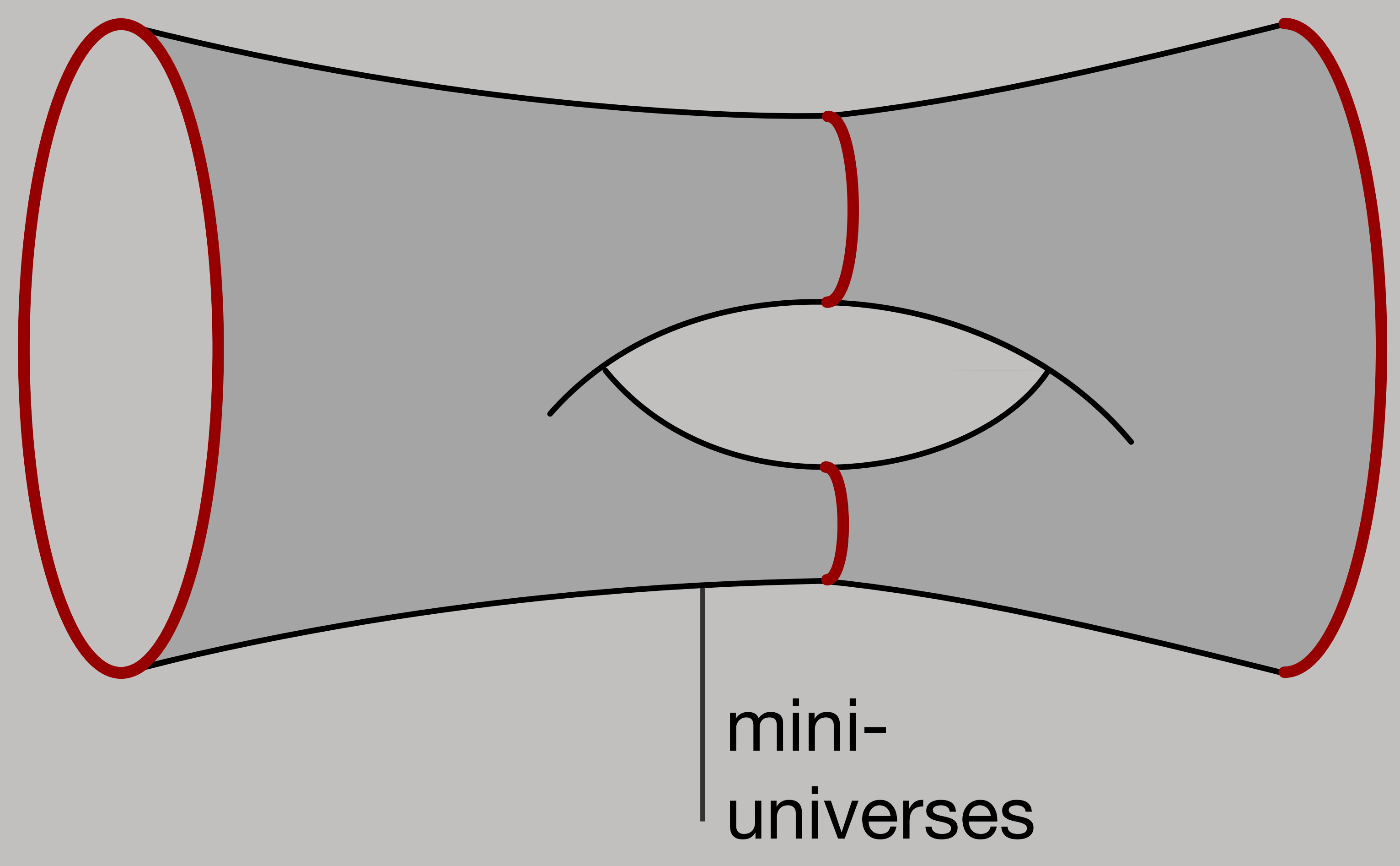
KS-field theory

Kodaira-Spencer field theory

Kodaira, Spencer 58
Vafa *et al. 94*
Post *et al. 22*

Aka the universe string field theory of JT gravity

Think of JT partition sum as asymptotic expansion of some quantum theory



Need quantum field theory of scattering of universes or closed strings →

Kodaira-Spencer (KS) field theory aka mini-universe field theory.

What is KS theory?

- KS: $2d$ CFT of chiral boson with with cubic nonlinearity

Post *et al.* 22

- theory defined on Riemann surface S_{JT} with cut singularity encoding DoS: the spectral curve

- perturbation theory in nonlinearity produces JT partition sum.

- conceptually: a string field theory

Introducing boundary sources (flavor probe branes) and taking low energy limit we map onto effective theory of quantum chaos

aa, Sonner, *et al.* 22

Chaos from KS

Consider correlation function

$$\left\langle e^{+\phi(\epsilon_1^+)}\, e^{-\phi(\epsilon_2^+)}\, e^{+\phi(\epsilon_1^-)}\, e^{-\phi(\epsilon_2^-)} \right\rangle_{\mathrm{KS}}$$

- CFT perspective: $e^{\phi(z)}$ is primary field
- Chaos perspective: $e^{\pm\phi(z)}$ holographically dual to $\det(z-H)^{\pm 1}$
- String theory perspective: $e^{\pm\phi(z)}$ insertion of compact/non-compact probe brane at energy z.

Chaos from KS

With $(\epsilon_1^+, \ldots, \epsilon_2^-) \equiv (x_1, \ldots, x_4) \equiv X$,

$$\left\langle e^{+\phi(x_1)}\, e^{-\phi(x_2)}\, e^{+\phi(x_3)}\, e^{-\phi(x_4)} \right\rangle_{\mathrm{KS}}$$

energy-
like

super Vandermonde

normal ordering

$$= \int dY\, e^{\frac{i}{\lambda} X\cdot Y}\, \Delta(Y) \left\langle :\, e^{+\phi(y_1)}\, e^{-\phi(y_2)}\, e^{+\phi(y_3)}\, e^{-\phi(y_4)} \,: \right\rangle_{\mathrm{KS}}$$

diagonal elements
of supermatrix

$$= \int dY\, e^{\frac{i}{\lambda} X\cdot Y}\, \Delta(Y)\, e^{-\Gamma(Y)}$$

Chaos from KS

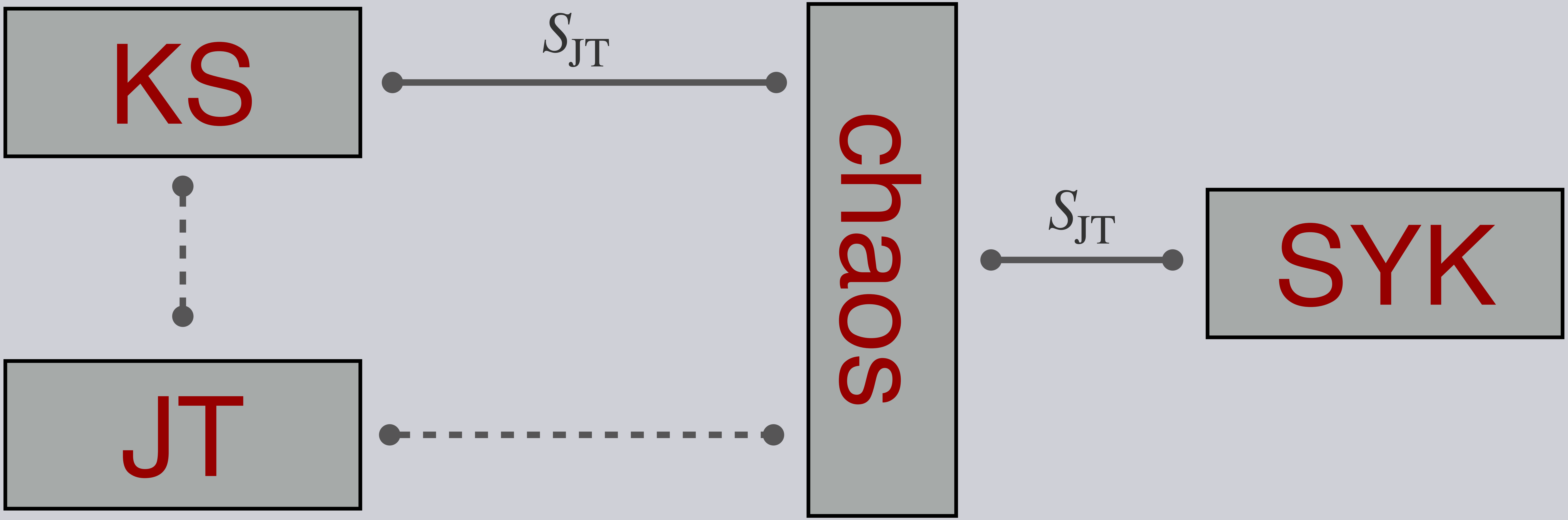
$$\left\langle : e^{+\phi(x_1)}\, e^{-\phi(x_2)}\, e^{+\phi(x_3)}\, e^{-\phi(x_4)} : \right\rangle_{\mathrm{KS}} = \int_{\mathrm{GL}(2|2)} dA\, e^{\lambda^{-1}(i\,\mathrm{str}(AX) - \Gamma(A))}$$

$$\longrightarrow \quad \boxed{\int_{\mathrm{A}_{2|2}} dQ\, e^{\frac{i}{\lambda}\,\mathrm{str}(QX)}}$$

Efetov 81

- (Statistics of) micro-spectrum fully resolved
- non-perturbative duality to SYK
- a chaotic string theory
- geometric signatures of chaos field theory — Altshuler-Andreev saddles, non-compactness, ⋯ — afford string interpretation (not discussed here.)

conclusions



perturbation theory/info on μ-structure lost

non-perturbative